Engineering Notes

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Static Pressure in the Slipstream of a Propeller

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Introduction

IN some books and papers,[1-3] the suggestion is made that according to the theory, the static pressure in the slipstream of an active propeller, even far downstream, would exceed the static pressure in the fluid outside of the slipstream. In the opinion of the author this suggestion is incorrect and based on an improper use of the linearized flat model of the trailing vortex sheet of a wing. In this Note, a consistent theory is developed which yields the more plausible result that the static pressure in the slipstream tends to the freestream pressure.

Consider the flow model for a lifting wing, with a flat trailing vortex sheet moving down with velocity W far behind the wing, as introduced by Prandtl.[4] This model is very appropriate for the first approximate computation of the induced velocities at the wing. The rolling up of the vortex sheet in reality proceeds so gently that its influence at the wing is felt only as a higher order effect. It should be understood, however, that the approximation of this rolling-up discontinuity sheet by a flat one gives rise to an inconsistency at the edge. The sheet can only be forced to stay flat by neutralizing the lateral suction force F (Fig. 1a) at the edge. The sheet in reality starts rolling up behind a wing due to the absence of a structure capable to sustain that force (see Ref. 5). The rolling up freely continues until the trailing vorticity is more or less concentrated in two vortices in the "centers of gravity" of the vorticity of the once flat sheet halves (Fig. 1b). These vortices move, under each others influence, downward with a velocity W^* much smaller than the velocity W which a flat sheet with suction forces would have.

Adapting the flat trailing vortex sheet model to a propeller with a finite number of blades moving at a finite velocity, Betz[6] moulded the trailing vortex sheets into helically wound surfaces. In an appendix to Betz's paper, Prandt[6a] introduced a two-dimensional semi-infinite model for the outer region of the slipstream. These slipstream models with vortex sheets have proved to be useful for various effective propeller computations.[7,8] However, extending the use of the sheet model to the computation of pressures in the slipstream, as is done by, e.g., Theodorsen[1] and Bramwell[2,3] leads to the erroneous result that the static pressure in the slipstream is $\rho W^2/2$ higher than the freestream static pressure. This result can be inferred immediately when we consider Prandtl's semi-infinite model in a frame of reference fixed to the vortex sheets (see Fig. 2).

The outer flow W has static pressure p_1. The streamlines of the outer flow W enter and leave the slipstream zone through the "boundary" of the slipstream. As the flow comes to rest between the sheets the static pressure p_{s1} in the slipstream rises to the stagnation pressure p_{01} of the freestream, so that $p_{s1}=p_{01}=p_1+\rho W^2/2$. The subscript s stands for slipstream, 0 for stagnation condition while 1 refers to a situation with the frame of reference fixed to the sheets, a subscript 2 will refer to a frame fixed to the freestream. At the edges of the vortex sheets there are singularities in the velocity of the form $\mathrm{d}\chi/\mathrm{d}z=-iW(a/2\pi z)^{1/2}$, provoking suction forces in x-direction $F=\rho W^2 a/2$.

Considering the above model in a frame of reference fixed to the external medium, which is at rest, the vortex sheets are moving downward with velocity W and the problem must be treated as an instationary one. In the external medium, far from the edges, the static pressure p_2 is identical to the stagnation pressure p_{02}. The complex potential of the flow now is

$$\chi=-(Wa/\pi)\operatorname{arccos} e^{(\pi/a)(z+iWt)}+iW(z+iWt)$$

leading to

$$\frac{\mathrm{d}\chi}{\mathrm{d}z}=\frac{-iW}{(1-e^{-(2\pi/a)(z+iWt)})^{1/2}}+iW$$

so that $\mathrm{d}\chi/\mathrm{d}z\rightarrow iW$ for $x\rightarrow-\infty$ and $\mathrm{d}\chi/\mathrm{d}z\rightarrow 0$ for $x\rightarrow+\infty$.

The static pressure now follows from Bernoulli's law for instationary flow

$$p=p_0-\frac{\rho}{2}\left|\frac{\mathrm{d}\chi}{\mathrm{d}z}\right|^2-\rho\frac{\partial\phi}{\partial t} \tag{1}$$

For the static pressure p_{s2} in the slipstream we need the value of $\partial\phi/\partial t$ which we compute† from

$$\frac{\partial\phi}{\partial t}=\mathrm{Re}\left(\frac{\partial\chi}{\partial t}\right)=\mathrm{Re}\left(\frac{\mathrm{d}\chi}{\mathrm{d}z^*}\frac{\partial z^*}{\partial t}\right)=-W^2$$

where $z^*=z+iWt$.

Inserting the values of $\mathrm{d}\chi/\mathrm{d}z$ and $\partial\phi/\partial t$ in Eq. (1) yields a result equivalent to the stationary case

$$p_{s2}=p_{02}+\rho W^2/2=p_2+\rho W^2/2$$

†A more illustrative way to find the value of $\partial\phi/\partial t$ is to consider ϕ in an observation point far to the left of the edges, where the potential jumps with $\Delta\phi=\Gamma$ every time a sheet passes from above. Here Γ is understood to have the value

$$\Gamma=-\int_{-\infty}^{0}\gamma(x)\,\mathrm{d}x=Wa$$

Between the sheets ϕ is distributed as

$$\phi=\int\frac{\partial\phi}{\partial y}\,\mathrm{d}y=-W(y+Wt)$$

The potential distribution is a sawtooth line traveling in negative y direction (Fig. 4).

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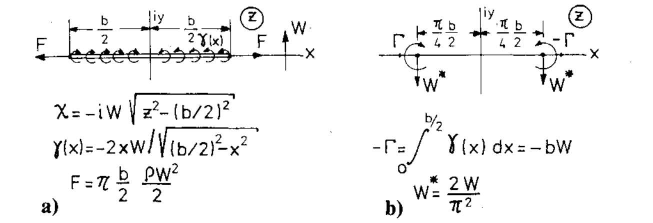

Fig. 1 a) Flat sheet of span b in normal flow W; b) Concentrated vortices Γ moving forcefree at velocity W^*.

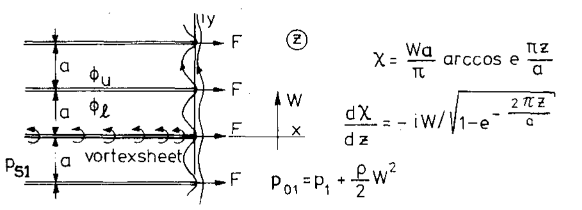

Fig. 2 Semi-infinite flat sheet model of the slipstream.

We find that in the flat sheet model the static pressure p_s in the slipstream exceeds the static pressure p in the external medium by $\rho W^2/2$, whether we consider the model stationary or moving. This high static pressure is inherent to this model of the slipstream with stretched vortex sheets having suction forces on their edges. The total head in the slipstream is in the stationary case p_{0s1}, equal to the total head p_{01} of the outer flow. In the instationary case we must include $\partial\phi/\partial t$ in the total head, yielding the higher value

$$p_{0s2}=p_{02}-\rho\frac{\partial\phi}{\partial t}=p_2+\frac{\rho}{2}\left|\frac{\mathrm{d}\chi}{\mathrm{d}z}\right|^2=p_{02}+\frac{\rho}{2}W^2$$

In a realistic model of the developed slipstream the vortex sheets have rolled up and the resulting vortices of course are free to move with the local flowfield as it is generated by the other free vortices. When the slipstream has velocity W, the vortices move with velocity $W/2$. The sheets in the previous model were vorticity sheets sustaining a jump $\Delta\phi=\phi_u-\phi_l$ in ϕ varying from $\Delta\phi=0$ at the edges to $\Delta\phi=Wa$ far away from the edges. Now with concentrated vortices, provoking a discontinuity in ϕ equal to the strength of the vortices, $\Delta\phi=Wa$, the discontinuities in ϕ can be thought to occur across cuts in the z plane between vortex pairs. These cuts move with the vortices with velocity $W/2$. Now, whether we move with the wake or with the external medium, the situation is instationary in either case. We consider a two-dimensional model of the slipstream with rolled up vortex sheets from a frame of reference which is fixed to the external medium at rest, (see Fig. 3). The wake moves downward with velocity W.

It should be noted that the velocity $\Gamma/2a=-W/2$ of the row at $x=b/2$, subsequently $x=-b/2$ is the velocity induced by the vortices at $x=-b/2$, subsequently $x=b/2$. The vortices are moving forcefree. The static pressure follows from Eq. (1) as before but now in the slipstream we find

$$\frac{\partial\phi}{\partial t}=\mathrm{Re}\left(\frac{\mathrm{d}\chi}{\mathrm{d}z^*}\frac{\partial z^*}{\partial t}\right)=-\frac{W^2}{2}$$

where $z^*=z+i(W/2)t$, yielding a static pressure p_s in the

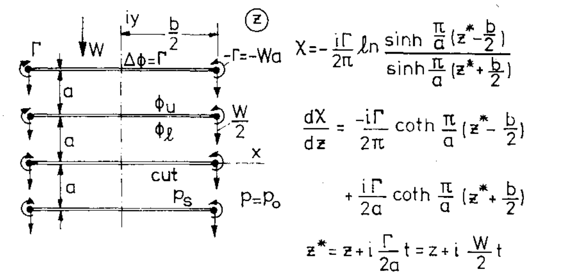

Fig. 3 A two-dimensional model of the slipstream with free vortices.

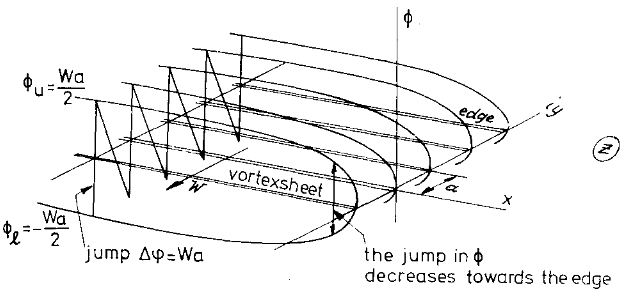

Fig. 4 Potential distribution traveling in negative y direction.

slipstream

$$p_s=p_0-\frac{\rho}{2}W^2+\frac{\rho}{2}W^2=p_0=p$$

The total head p_{0s} in the slipstream now is, including the $\partial\phi/\partial t$ term,

$$p_{0s}=p_0-\rho\frac{\partial\phi}{\partial t}=p_s+\frac{\rho}{2}\left|\frac{\mathrm{d}\chi}{\mathrm{d}z}\right|^2=p_0+\frac{\rho}{2}W^2$$

It seems to the author that these are the correct results for p_s and p_{0s} and that the vortex sheet model should not be used for the computation of pressures in the slipstream.

For completeness we should mention that the situation of Fig. 3, considered in a frame moving with the vortices yields a stationary model. As the vortices move with the mean of the velocities inside and outside the wake, the situation will be antisymmetric considered locally from a vortex row. It will be clear at once that the pressure on both sides of the vortex rows has the value

$$p=p_0-(\rho/2)(W/2)^2$$

It is also clear that in every frame of reference moving parallel to the straight vortex rows the static pressure will be uninfluenced but differences in total head inside and outside the wake will be manifest.

As a final remark we might add that W.R. Sears in his Note on induced drag and conditions downstream of a lifting wing,[9] using the flat trailing vortex sheet model, concludes to overpressures in the wake where a more realistic horseshoe vortex model would lead to the more plausible result of low pressures in the Trefftz plane.

Acknowledgment

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Design Considerations for Duty Cycle, Life, and Reliability of Small Limited-Life Engines

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Nomenclature

a	= level of confidence
MTBF	= mean time between failures
R	= reliability
r	= failures
χ	= chi-square distribution

Introduction

DESIGN requirements, design criteria and verification methods related to preliminary design and program planning for small limited life turbine engines have significant differences from other engine technologies. Operating design life of components is defined by development test requirements, rather than mission life requirements. Interpolation between operating mission limits is required to demonstrate operating life for limited life engines rather than extrapolation of accelerated testing required for long life.

Storage life requirements are very stringent. The correlation of reliability prediction vs accelerated verification testing for engines requiring extended dormant storage is being validated.

In this discussion, I have limited this definition of "small limited-life engines" to small gas turbine engines typically applied as emergency power units for flight or ground power generation, target or reconnaissance drone propulsion, or cruise missile propulsion.

These applications have unique requirements which cause modifications in the usual methods of design, design verification, and reliability prediction used for turbomachinery. The unique requirements are as follows: 1) short operating mission life—the actual mission can be as short as 1 cycle and ½ hour in duration; 2) relatively low cost; 3) long storage life—vehicle may be stored up to 10 years in an adverse environment; 4) minimum storage and preoperational maintenance dictated by life cycle cost considerations; 5) very high starting and operating reliability required after extended storage; 6) very short times required from start initiation to full power. This last requirement applies to cruise missiles launched from a terrain following carrier or an emergency power unit where power interruption is critical.

Each of these requirements leads to variations in the usual engine design criteria.

Operating Design Life

The operating design life for components for this type of engine is not defined by mission length. To establish reasonable design lives, the component should be divided into four categories. These are 1) high cost components; 2) critical components (critical in terms of engine or component performance); 3) parts normally discarded at disassembly (gaskets, seals, etc.); 4) other.

Design life for each of these categories is calculated by obtaining the life requirement based on the following four conditions.

a) Maximum operating time and cycles prior to engine disassembly: This must include the maximum hours accumulated during engine checkout prior to use, as this time may exceed the mission operating life. This value is the minimum applied to all categories of parts.

b) Total operating time and cycles for the design life of the engine: Normally this is the maximum number of overhaul periods multiplied by the value from a. This life is the minimum value to be applied to components in categories 1, 2, and 4.

c) Cost effective life for engine development and qualification testing: This value is calculated by totaling the test hours planned for engine development, also the total time required for qualification and dividing by number of engines assigned to these tasks. These values are used to establish life goals for components in category 1. Conversely, attainable life will determine the number of engines and spares required for development and qualification testing. During the full-scale development phase of F107 family of cruise missile engines, 3157 test hours were run with 14 engines; or 225 hrs per engine. During the qualification phase, 24 engines were built and 387 hrs of testing with 693 starts were accomplished. Therefore the average engine required 16 hrs of operation and 29 starts. Maximum engine time was 20 hrs during qualification. This effort was required to support an actual mission requirement of one start and 5 hrs of operation. The design life for components in this case ranged from 10 to 50 times the mission requirement for economical development and qualification testing. Variation within these limits is determined on a component by component basis based on cost, weight, and anticipated performance loss.

d) The purpose of extended design life on this case is to provide sufficient component usable life to allow reliable performance measurements to evaluate changes which require the use of "back-to-back" test. Small performance or life differences became difficult to evaluate when engine deterioration or build-to-build variations exceed the values being measured. This requirement applied to category 2 (critical) components. A life requirement of three times the mission would allow evaluation of two variables plus a baseline run in one test series requiring an entire mission simulation.

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